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DETERMINATION OF PIPE STRESSES

IN INDUSTRIAL PLANTS

A THESIS

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by

Roy Lamar Cash

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DETERMINATION OF PIPE STRESSES IN INDUSTRIAL PLANTS

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SUMMARY

Piping is an important component in the design of chemical and other industrial plants. Among the engineering principles involved in successful plant design is pipe stress analysis. The present-day designer has available many methods for determining the magnitude of pipe stresses. The intent of this study was to aid the designer by reviewing the principal stresses which might be present in piping at the design conditions, and by discussing the methods which are available for determining one of the most prevalent of these designated stresses, the thermal stress due to operation at elevated temperature.

Various methods for the analysis of thermal stress were first grouped into four general types: analytical, standardized shapes, graphical-analytical, and graphical. Then, one typical method from each group was selected, described in detail, and employed in the analysis of the thermal stress in each of three piping configurations: angle bend, unbalanced loop, and three-dimensional bend. Each method was illustrated by the procedure required for the particular method used. A summary was made of the results obtained from the use of the four methods in analyzing the three bends, and the various values for the thermal stress were compared.

It was noted that the analytical type of analysis was used as the basis of two of the other types investigated: standardized shapes and graphical-analytical. The time required for the various methods of

analysis, arranged in descending order of magnitude, was as follows: analytical, graphical, graphical-analytical, and standardized shapes. It was observed that the analytical and graphical types afforded more opportunity for a close check to be methodically applied during the analysis, while the standardized shapes and graphical-analytical methods required the use of values which could not be readily checked by use of the data given. The scope of this study was confined to the description, classification, illustration, and comparison of various methods of analysis, and a detailed check on the accuracy of the individual results was not attempted. For a more comprehensive investigation of the relative accuracy of the four types of analysis, a larger variety of configurations and design conditions should be used. Such results might then be compared with actual stress values pre-determined from laboratory tests or other accurate methods.

CHAPTER I

INTRODUCTION

Importance of Pipe Stress Analysis.--Pipe failures due to excessive stresses cause plant shutdowns, loss of production, injuries to personnel, and damage to the reputation of the designer.

During the design of an installation involving complex and expensive equipment, there is sometimes a tendency on the part of the designer to place most of the emphasis upon the equipment and process, and insufficient effort upon piping design (1). This practice is often due to the designer's lack of knowledge regarding the principles of piping design, and the dependance of a plant upon piping for successful operational results.

Early Treatment of Pipe Stress Investigation.--The systemmatic study of stresses in piping began in 1910 with the work of Bantlin (2), who reported that there was greater flexibility in a pipe bend due to the tendency of the circular cross section to assume an oval shape.

A year later, Karman (2) investigated the distribution of stresses during the bending of curved tubes. During the following thirty-five years, such workers as Hovgaard, Karl, and Vigness (2) established the fact that the variation in longitudinal stresses for a curved pipe during bending is not linear, and that the actual value of the stresses may be calculated by applying a "stress intensification factor" to the stress value obtained by considering the pipe to be straight instead of curved. In 1934, Tingey (4) introduced the elastic center theory to stress analysis, and a year later Spielvogel and Kameros (5), (6) improved upon this method of attack by the use of conjugate axes in calculations. In 1929, Shipman (3) applied simple mathematics to the analysis of single-plane bends, and developed a relatively simple method of stress calculation by the use of sign conventions and shape coefficients.

Modern Methods of Analysis in Plant Design.--Before 1935, published work on the subject of pipe stress analysis had been based upon piping in a single plane only. However, in that year, Hovgaard (7) applied the elastic center and conjugate axes techniques to the analysis of two-plane piping. In 1939, Walker and Crocker (8) published a combination graphical-analytical method of analysis. In 1941, The M. W. Kellogg Company (9) produced a general analytical method which could be applied to both single- and two-plane systems. In 1943, Spielvogel (10) applied the elastic center to both types of systems in an analytical method similar to that of the Kellogg Co.

In 1946, Fish (11) published a graphical method of analysis as a means of rapidly analyzing a piping system for stresses. Spielvogel (12), in 1955, furnished improvements to his previously published analytical method. In 1956, The M. W. Kellogg Company (13) published a second edition of the general analytical method and included shape coefficients and standardized calculation forms.

<u>Purpose of Study</u>.--Apparently there has been no previous summary of pipe stress analysis methods treated from the viewpoint of the chemical engineer. From the foregoing brief history of the development of pipe analysis methods, it may be apparent that work on analysis techniques has proceeded along several paths. It is the purpose of this study to discuss, illustrate, and compare the analytical, standardized shapes, graphical-analytical, and graphical methods of pipe stress analysis, so that the engineer may have a clear presentation of these different systems of analysis.

CHAPTER II

PIPE STRESS ANALYSIS

In planning for the design of a new installation, the chemical engineer is interested in achieving a completed design that has the highest production capacity and the minimum amount of maintenance consistent with good practice for the type of process involved. The chief components of a typical plant might be listed as building and grounds, equipment, piping, instrumentation, and electrical. Of these five items, piping is often as important as any of the others.

Many modern chemical processes operate at elevated temperatures and pressures which induce stresses in the process piping. In designing for this type of chemical operation, the chemical engineer should use reliable and economical methods for determining the pipe stresses.

Procedure Employed in Reviewing Available Methods.--The method of investigation of material for this study consisted of a literature survey for information on pipe stress analysis, followed by a selection of basic types of analysis involved, and the application of these basic methods to each of three typical piping configurations in order to illustrate their use and obtain a comparison between the results. No apparatus or equipment was required for this type of approach.

Factors Affecting the Selection of the Method to be Used.--In applying design principles to engineering work, the chemical engineer often finds that more than one method of design may be used, and a selection must be made from among the various paths of attack. This condition presents itself to the designer of piping for a chemical plant. The degree of accuracy required must be balanced with the relative cost of the engineering work in the design. In addition, the availability of time for the design work must be considered in coordinating this part of the work with other phases of the plant design.

CHAPTER III

DETAILED DISCUSSION OF PIPE STRESS ANALYSIS METHODS

Basic Considerations in Stress Analysis.--In the analysis of piping for stresses, basic considerations involve an accurate determination of the piping configuration to be investigated, location of anchors, position of take-off lines, location and type of hangers between anchors, accurate information regarding design conditions, and a knowledge of the various types of stresses to be considered in the analysis. Since the scope of this study covers only stresses, a discussion of basic factors concerning pipe stresses is included in this chapter.

The various types of stresses which might be present in the piping of a chemical plant are as follows:

(1) Longitudinal stress due to internal pressure (14), for pipe closed at both ends = $S_2 = \frac{pd}{4t}$

where S_2 = stress, psi., p = internal pressure, psig., d = inside diameter of pipe, in., t = wall thickness of pipe.

(2) Tensile stress due to internal pressure (15) = $S_t = \frac{pr}{t}$

where S_t = stress, psi., p = internal pressure, psig., r = inside radius of pipe, in., t = wall thickness.

(3) Bending stress due to weight between pipe supports (16) = S_w

$$S_w = \frac{1.5\ wL + 1.2\ w\ L^2}{S_m} \quad \text{(basis three spans)}$$

where S_w = psi., w = weight of pipe, covering and contents, lb./ft.; S_m = section modulus for the pipe, inches^3, L = length between supports, ft.

(4) Torsional stress due to tendency to rotate (17) = $S_T = \frac{M_t}{2\ Z}$

where S_T = psi., M_T = tortional bending moment, inch/lb.; Z = section modulus, inches^3.

(5) Resultant longitudinal stress due to bending from thermal expansion (18) = S_b, calculated from pipe stress analysis.

(6) Combined stress due to thermal expansion (18) = S_E = $\sqrt{S_b^{\,2} + 4S_T^{\,2}}$

Since the various methods of stress analysis under consideration apply to stresses resulting from elevated temperature, the comparison of their values will be based upon the value of S_b, the bending stress due to thermal expansion. Also, the value of S_b will be calculated at the operating design temperature, rather than for the cold condition.

(7) Other possible stresses in piping are due to (18):

Additional loads on piping span

Wind

Earth tremors

Special shock loading

Unbalanced static pressure or flow effects

Vibration

Comparison Between Types of Analysis.--Although there are several methods which might be used for each type of analysis, one method has been selected for each type, as follows:

Type	Method Selected
Analytical	A - S. W. Spielvogel
Standardized Shapes	B - Grinnell Company
Graphical-Analytical	C - Tube Turns
Graphical	D - National Valve & Mfg. Co.

The analytical method makes use of the principles that the sum of all horizontal and vertical forces is equal to zero, and that the sum of all moments about a fixed point is equal to zero. Three equations are obtained by considering the distortion in the system due to restrictions which prevent the expansion of the pipe. These equations are simplified to two relationships by the assumption that one of the two supports is released and temporarily connected to a rigid bracket leading to the center of gravity of the lines, for a single-plane piping system. For a two-plane system, the projection of the piping in each of three planes, X-Y, Y-Z, and X-Z, is considered, and three equations with three unknowns are developed for solution and determination of forces and stresses in the system (see the Appendix for a more detailed discussion of this method).

The standardized shapes method involves the use of predetermined factors resulting from the analytical method applied to the standardized shape under consideration. By use of such factors with dimensions of the system and pipe properties, the bending stress, as well as torsional stress, may be readily determined.

The graphical-analytical method makes use of predetermined graphs having curve values corresponding to the dimensions and properties of the piping. The curve values have previously been calculated by an analytical method. Substitution of the values obtained from the graphs and charts into equations produces a value for the stress due to thermal expansion.

The graphical method involves the plotting of points corresponding to the lengths of pipe in the piping system, the determination of a neutral axis for the piping system, location of the intersection of the neutral axis with each length of pipe, the plotting of neutral axis points, connection of the plotted points to form a moment area, and substitution of the value of the area into equations which produce a value for the thermal stress.

From the above discussion it is seen that the analytical method has been used as the basis for the factors used in two of the other methods.

CHAPTER IV

COMPARATIVE RESULTS FROM VARIOUS METHODS OF ANALYSIS

The results obtained from the use of each of the four methods of analysis and the three piping configurations are shown on pages 11 through 28.

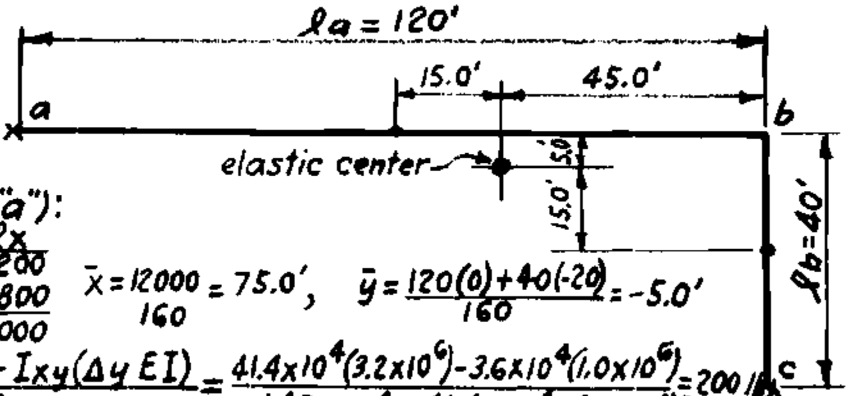
<u>Tabulated Results for Three Typical Configurations</u>.--A summary of the results from the stress analyses is as follows:

Table 1. Thermal Stress Values

Method Used	Configuration	Maximum Thermal Stress, psi.
A - S. W. Spielvogel	Angle Bend	9510
B - Grinnell Company	Angle Bend	11400
C - Tube Turns	Angle Bend	12100
D - Nat'l. Valve & Mfg. Co.	Angle Bend	9400
A - S. W. Spielvogel	Unbalanced Loop	2680
B - Grinnell Company	Unbalanced Loop	3100
C - Tube Turns	Unbalanced Loop	2000
D - Nat'l. Valve & Mfg. Co.	Unbalanced Loop	2060
A - S. W. Spielvogel	Three-Dimensional Bend	1030
B - Grinnell Company	Three-Dimensional Bend	3030
C - Tube Turns	Three-Dimensional Bend	----
D - Nat'l. Valve & Mfg. Co.	Three-Dimensional Bend	1640

Stress Analysis for 6" Schedule 80, ASTM A-53 Grade B Pipe
Design Conditions: 200 psig., 550°F.

Type 1, Method A,
S. W. Spielvogel
(21)
Elastic Center



Centroid (basis "a"):

	ℓ	x	ℓx
ab	120	60	7200
bc	40	120	4800
	160		12000

$$\bar{x} = \frac{12000}{160} = 75.0', \quad \bar{y} = \frac{120(0)+40(-20)}{160} = -5.0'$$

$$F_x = \frac{I_y(\Delta x EI) + I_{xy}(\Delta y EI)}{I_x I_y - I_{xy}^2} = \frac{41.4\times10^4(3.2\times10^6) - 3.6\times10^4(1.0\times10^6)}{1.49\times10^4 \times 41.4\times10^4 - (3.6\times10^4)^2} = 200 \text{ lb.}$$

$$F_y = \frac{I_x(\Delta y EI) + I_{xy}(\Delta x EI)}{I_x I_y - I_{xy}^2} = \frac{1.49\times10^4(1.0\times10^6) - 3.6\times10^4(3.2\times10^6)}{1.49\times10^4 \times 41.4\times10^4 - (3.6\times10^4)^2} = 15.8 \text{ lb.}$$

length	ℓ'	x'	y'	ℓxy
ab	120	-15.0	+45.0	-9000
bc	40	+45.0	-15.0	-27000
			I_{xy} =	36000

$$120 \times 5 = 600 \;(I_x)$$
$$\frac{40^3}{12} + 40(15)^2 = 14350$$
$$I_x = 14950$$

$$\frac{120^3}{12} + 120(15)^2 = 171000 \;(I_y)$$
$$120(45.0)^2 = 243000$$
$$I_y = 414000$$

$$\Delta x EI = \frac{4.11}{100}\frac{(120)}{12} 27\times10^6 \times 12^2 \times \frac{40.49}{12^4} = 3.2\times10^6, \quad M = -200(5.0) + 15.8(45.0)$$

$$\Delta y EI = \frac{4.11}{100}\frac{(40)}{12} 27\times10^6 \times 12^2 \times \frac{40.49}{12^4} = 1.0\times10^6, \quad M = +6110 \text{ lb.ft.}$$

$$S_b = \beta M/S_m = 1.64(6110)(12)/12.23 = 9{,}510 \text{ psi.}$$

Figure 1. Angle Bend, Type 1, Method A

Type 2, Method A,
S.W. Spielvogel (22)
Elastic Center

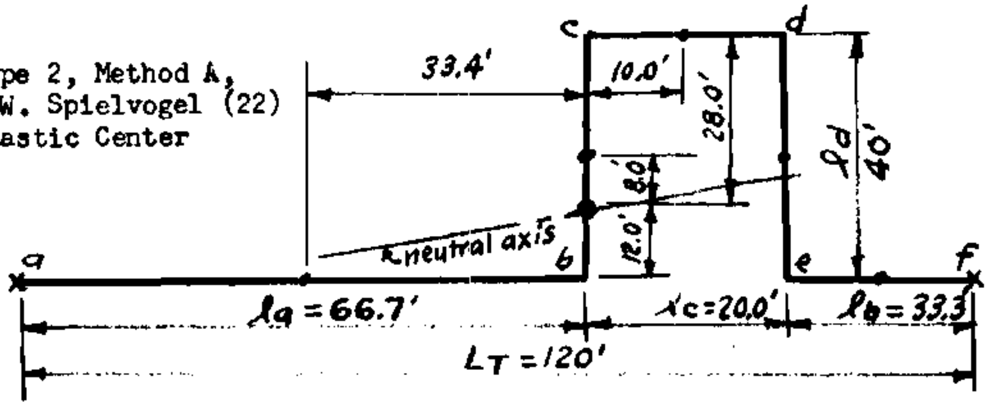


Figure 2. Unbalanced Loop, Type 2, Method A

THERMAL STRESS CALCULATIONS
UNBALANCED LOOP, TYPE 2, METHOD A
SPIELVOGEL, ELASTIC CENTER
REFERENCE FIGURE 2

Location of centroid (basis a):

	ℓ	x'	$\ell x'$	y'	$\ell y'$
ab	66.7	33.4	2228	0	0
bc	40.0	66.7	2668	20	800
cd	20.0	76.7	1534	40	800
de	40.0	86.7	3468	20	800
ef	33.3	103.4	3443	0	0
	200.0		13341		2400

$\bar{x} = \frac{13341}{200} = 66.7'$ $\qquad \bar{y} = \frac{2400}{200} = 12.0'$ $\qquad I_o = \frac{\ell^3}{12}$

Branch	ℓ	x	y	$\ell x^2 + I_o$	$\ell y^2 + I_o$	I_{xy}
ab	66.7	-33.4	-12.0	77,437 24,729	9,605 0	+26,730
bc	40.0	0	+ 8.0	0 0	2,560 5,800	0
cd	20.0	+10.0	+28.0	2,000 667	15,680 0	+ 5,600
de	40.0	+20.0	+ 8.0	16,000 0	2,560 5,800	+ 6,400
ef	33.3	+50.0	-12.0	83,250 3,077	4,795 0	-19,980

$I_y = 224,160$

$I_x = 46,800 \qquad I_{xy} = +18,750$

$$I_xF_x - I_{xy}F_y = \Delta X\, EI_p$$

$$-I_{xy}F_x + I_yF_y = \Delta y\, EI_p$$

$$F_x = \frac{I_y(\Delta X\, EI_p) + I_{xy}(\Delta y\, EI_p)}{I_xI_y - I_{xy}^2},$$

$$F_y = \frac{I_x(\Delta y\, EI_p) + I_{xy}(\Delta X\, EI_p)}{I_xI_y - I_{xy}^2}$$

THERMAL STRESS CALCULATIONS
UNBALANCED LOOP, TYPE 2, METHOD A
SPIELVOGEL, ELASTIC CENTER
REFERENCE FIGURE 2

$$\Delta x\ EI_p = \frac{4.11 \times 100}{100 \times 12} \times 27 \times 10^6 \times 12^2 \times \frac{40.49}{12^4} = 2.52 \times 10^6$$

$$\Delta y\ EI_p = \frac{4.11 \times 40}{100 \times 12} \times 27 \times 10^6 \times 12^2 \times \frac{40.49}{12^4} = 1.05 \times 10^6$$

$$F_x = \frac{2.24 \times 10^5(2.80 \times 10^6) + 1.88 \times 10^4(1.12 \times 10^6)}{4.68 \times 10^4 \times 22.4 \times 10^4 - (1.87 \times 10^4)^2} = 59.6 \text{ lb.}$$

$$F_y = \frac{4.68 \times 10^4(1.12 \times 10^6) + 1.88 \times 10^4(2.80 \times 10^6)}{4.68 \times 10^4 \times 22.4 \times 10^4 - (1.87 \times 10^4)^2} = 9.0 \text{ lb.}$$

From Figure 5, Point "C" is farthest from the neutral axis and therefore the point of maximum bending stress.

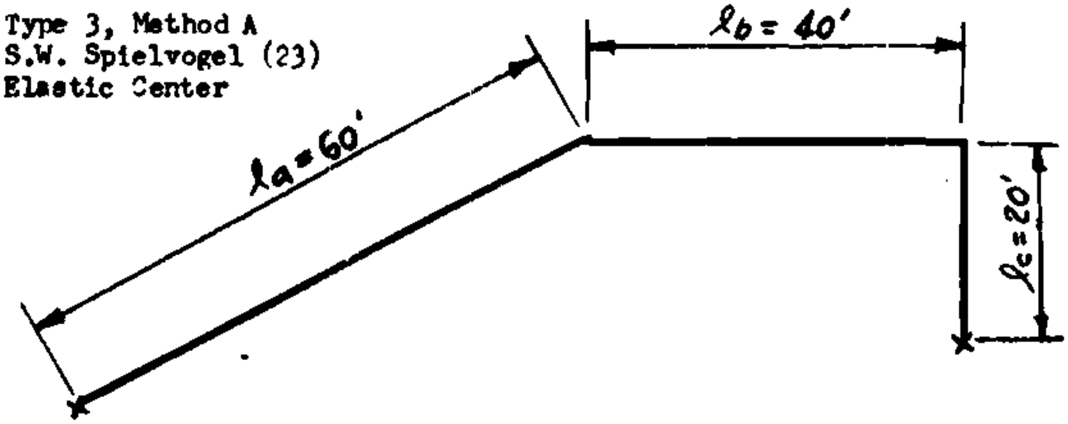
$$M_c = 9.0\ (0) + 59.6(28.0) = 1670 \text{ lb.ft.}$$

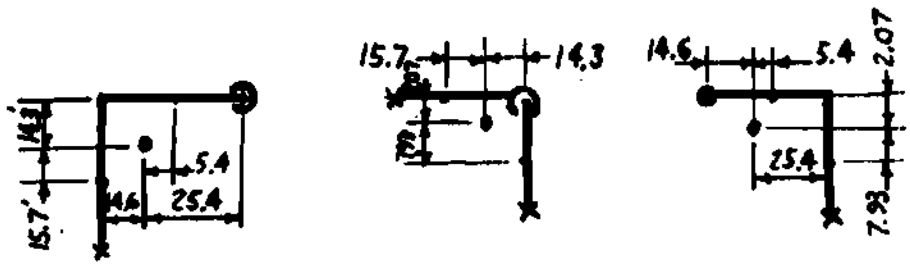
Bending stress at "C" due to thermal expansion = $S_c = \frac{\beta\ M_c}{S_m}$

$$S_c = \frac{1.64\ (1,670)\ 12}{12.28} = 2,680 \text{ psi.}$$

Stress Analysis for 6" Schedule 80, ASTM A-53 Grade B Pipe
Design Conditions: 200 psig., 550°F.

Type 3, Method A
S.W. Spielvogel (23)
Elastic Center





X-Y Plane　　Y-Z Plane　　X-Z Plane

$$\Delta X\,EI_p = \frac{4.11 \times 40}{100 \times 12} \times 27 \times 10^6 \times 12^2 \times \frac{40.49}{12^4} = 1.56 \times 10^6$$

$$\Delta Y\,EI_p = \frac{4.11 \times 60}{100 \times 12} \times 27 \times 10^6 \times 12^2 \times \frac{40.49}{12^4} = 1.04 \times 10^6$$

$$\Delta Z\,EI_p = \frac{4.11 \times 20}{100 \times 12} \times 27 \times 10^6 \times 12^2 \times \frac{40.49}{12^4} = 0.52 \times 10^6$$

Figure 3. Three-Dimensional Bend, Type 3, Method A

THERMAL STRESS CALCULATIONS
THREE-DIMENSIONAL BEND, TYPE 3, METHOD A
S. W. SPIELVOGEL, ELASTIC CENTER
REFERENCE FIGURE 3

Location of Centroid (Basis "d"):

X-Y Plane

Section	ℓ	x'	$\ell x'$	y'	$\ell y'$
de	60	0	0	-30	-1800
ef	40	-20	-800	-60	-2400
fg	1.3(20)	-40	-1040	-60	-1560
	126		-1840		-5760

$$\bar{x} = -\frac{1840}{126} = -14.6'$$

$$\bar{y} = -\frac{5760}{126} = -45.7'$$

Y-Z Plane

Section	y'	$\ell y'$	z'	$\ell z'$
de	-30	-1800	0	0
ef	-60	-2400	0	0
fg	-60	-1560	+10	+260
		-5760		+260

$$\bar{y} = \frac{-5760}{126} = -45.7'$$

$$\bar{z} = \frac{+260}{126} + 2.07'$$

X-Z Plane

Section	x'	$\ell x'$	z'	$\ell z'$
de	0	0	0	0
ef	-20	-800	0	0
fg	-40	-1040	+10	+260
		-1840		+260

$$\bar{x} = \frac{-1840}{126} = -14.6$$

$$\bar{z} = \frac{+260}{126} = +2.07'$$

THERMAL STRESS CALCULATIONS
THREE-DIMENSIONAL BEND, TYPE 3, METHOD A
S. W. SPIELVOGEL, ELASTIC CENTER
REFERENCE FIGURE 3

Calculation of Moments of Inertia, $in.^4$

Section	ℓ	x	y	$\ell x^2 + I_o$	$\ell y^2 + I_o$	ℓxy	Plane
de	60	+14.6	+15.7	12,800 0	14,800 18,000	+13,753	X-Y
ef	40	- 5.4	-14.3	1,165 5,340	8,190 0	+ 3,090	X-Y
fg	26	-25.4	-14.3	16,800 0	5,320 0	+ 9,350	X-Y

$I_y = 36,105$ $I_x = 46,310$ $I_{xy} = 26,193$

Section	ℓ	y	z	$\ell y^2 + I_o$	$\ell z^2 + I_o$	ℓyz	Plane
de	60	+15.7	- 2.07	14,800 18,000	257 0	- 1,950	Y-Z
ef	40	-14.3	- 2.07	8,190 0	171 0	+ 1,183	Y-Z
fg	26	-14.3	+ 7.93	5,320 0	1,635 1,464	- 2,950	Y-Z

$I_z = 46,310$ $I_y = 3,527$ $I_{yz} = -3,717$

Section	ℓ	x	z	$\ell x^2 + I_o$	$\ell z^2 + I_o$	ℓxz	Plane
de	60	+14.6	- 2.07	12,800 0	257 0	- 1,815	X-Z
ef	40	- 5.4	- 2.07	1,165 5,340	171 0	+ 447	X-Z
fg	26	-25.4	+ 7.93	16,800 0	1,630 1,464	- 5,230	X-Z

$I_z = 36,105$ $I_x = 3,522$ $I_{xz} = -6,598$

THERMAL STRESS CALCULATIONS
THREE-DIMENSIONAL BEND, TYPE 3, METHOD A
S. W. SPIELVOGEL, ELASTIC CENTER
REFERENCE FIGURE 3

Summary of Moments of Inertia:

I_x	I_y	I_z	I_{xy}	I_{yz}	I_{xz}
46,310	36,105	46,310	26,193	-3,717	-6,598
3,522	3,527	36,105			
49,832	39,632	82,415			

$$(1)\quad F_x I_x - F_y I_{xy} - F_z I_{xz} = \Delta x\, EI_p$$

$$(2)\quad -F_x I_{xy} + F_y I_y - F_z I_{yz} = \Delta y\, EI_p$$

$$(3)\quad -F_x I_{xz} - F_y I_{yz} + F_z I_z = \Delta z\, EI_p$$

$$(1)\quad +49{,}832\, F_x - 26{,}193\, F_y + 6{,}598\, F_z = 1.56 \times 10^6$$

$$(2)\quad -26{,}193\, F_x + 39{,}632\, F_y + 3{,}717\, F_z = 1.04 \times 10^6$$

$$(3)\quad +6{,}598\, F_x + 3{,}717\, F_y + 82{,}415\, F_z = 0.52 \times 10^6$$

$$(1)\quad F_x - 0.525\, F_y + 0.132\, F_z = 31.4$$

$$(2)\quad -F_x + 1.520\, F_y + 0.142\, F_z = 39.8$$

$$(4)\quad 0.995\, F_y + 0.274\, F_z = 71.2$$

$$(1)\quad F_x - 0.525\, F_y + 0.132\, F_z = 31.4$$

$$(3)\quad F_x + 0.564\, F_y + 12.490\, F_z = 79.0$$

$$(5)\quad 1.089\, F_y + 12.358\, F_z = 47.6$$

$$(5)\quad F_y + 11.200\, F_z = 43.8$$

$$(4)\quad F_y + 0.275\, F_z = 71.2$$

THERMAL STRESS CALCULATIONS
THREE-DIMENSIONAL BEND, TYPE 3, METHOD A
S. W. SPIELVOGEL, ELASTIC CENTER
REFERENCE FIGURE 3

(6) $10.925\ F_z = -27.4,\ F_z = -2.5$ lb.

(4) $F_y = 71.2 + .275(2.5) = 71.9$ lb.

(1) $F_x = 31.4 + .525(71.9) + .132(2.5 = 31.4 + 36.4 + .7 = 68.5$ lb.

Summary of Moments, lb. ft.

Point	X-Y Plane	Y-Z Plane	X-Z Plane
d	+68.5(45.7)-71.9(14.6)	-71.9(2.07)-2.5(45.7)	-68.5(2.07)+2.5(14.6)
	= +3140 - 1050	= -148 - 114	= -142 + 36
	= + 2090	= -262	= -106*
e	-68.5(14.3)-71.9(14.6)	-71.9(2.07)-2.5(14.3)	-68.5(2.07)+2.5(14.6)
	= -980 - 1050	= -148 - 36	= -142 + 36
	+ -2030	= -184*	= -106*
f	-68.5(14.3)+71.9(25.4)	-71.9(2.07)-2.5(14.3)	-68.5(2.07)-2.5(25.4)
	=-980 + 1830	= -148 - 36	= -142 - 61
	= +850*	= -184*	= -203
g	-68.5(14.3)+71.9(25.4)	+71.9(17.93)-2.5(14.7)	+68.5(17.93)-2.5(25.4)
	= -980 + 1830	= 1290 - 36	= 1230 - 61
	= +850*	= +1254	= -1169

*Torsional moment.

THERMAL STRESS CALCULATIONS
THREE-DIMENSIONAL BEND, TYPE 3, METHOD A
S. W. SPIELVOGEL, ELASTIC CENTER
REFERENCE FIGURE 3

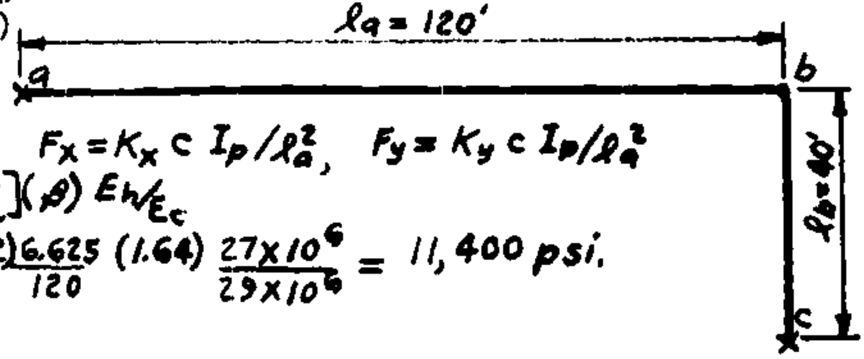
Point "d" has the maximum bending moment.

$$\text{Max. } S_b \text{ at "d"} = \frac{12\beta\sqrt{M_{xy}^2 + M_{yz}^2}}{S_m} = \frac{12(1.64)\sqrt{(2090)^2 + (262)^2}}{40.49}$$

$$\text{Max. } S_b = \frac{12(1.64)(2{,}110)}{40.49} = 1{,}030 \text{ psi.}$$

Stress Analysis for 6" Schedule 80, ASTM A-53 Grade B Pipe
Design Conditions: 200 psig., 550°F.

Type 1, Method B
Grinnell Co.(24)
Standardized
Shapes



$\ell_a/\ell_b = 3.0$, $F_x = K_x\, c\, I_p/\ell_a^2$, $F_y = K_y\, c\, I_p/\ell_a^2$

$S_b = [K_b\, c\, D/L](\beta)\, E_h/E_c$

$S_b = 209(642)\frac{6.625}{120}(1.64)\frac{27\times10^6}{29\times10^6} = 11,400$ psi.

Figure 4. Angle Bend, Type, 1, Method B

Type 2, Method B
Grinnell Co. (25)
Standardized
Shapes

$S_b = [K_b\, c\, D/L_T](\beta)\, E_h/E_c$

$S_b = [57.0(642)6.625/120](1.64)\frac{27\times10^6}{29\times10^6}$

$S_b = 3,100$ psi.

$\ell_a/\ell_b = 2.0$

$L_T/\ell_d = 3.0$

$L_T/\ell_c = 6.0$

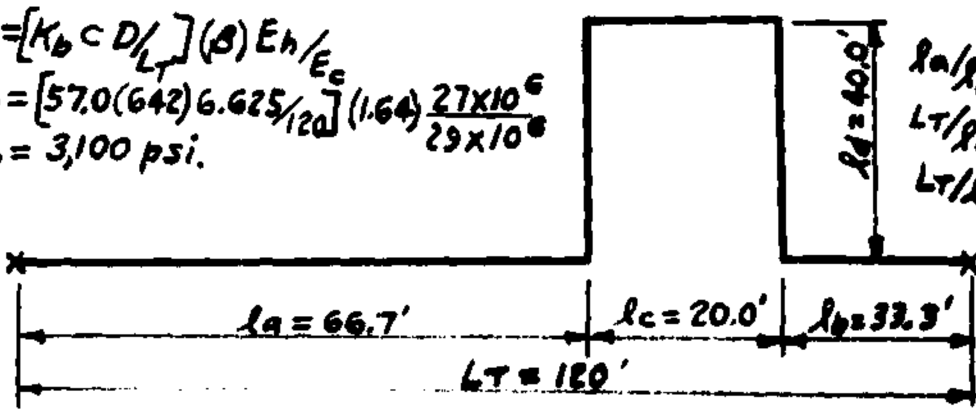
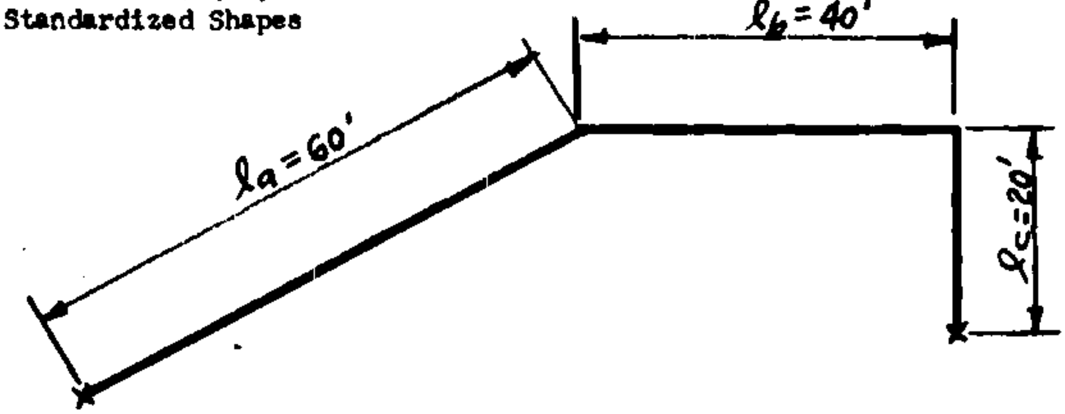


Figure 5. Unbalanced Loop, Type 2, Method B

Stress Analysis for 6" Schedule 80, ASTM A-53 Grade B Pipe
Design Conditions: 200 psig., 550°F.

Type 3, Method B,
Grinnell Co. (26)
Standardized Shapes



$\ell a/\ell_c = 3.0, \quad \ell b/\ell_c = 2.0$

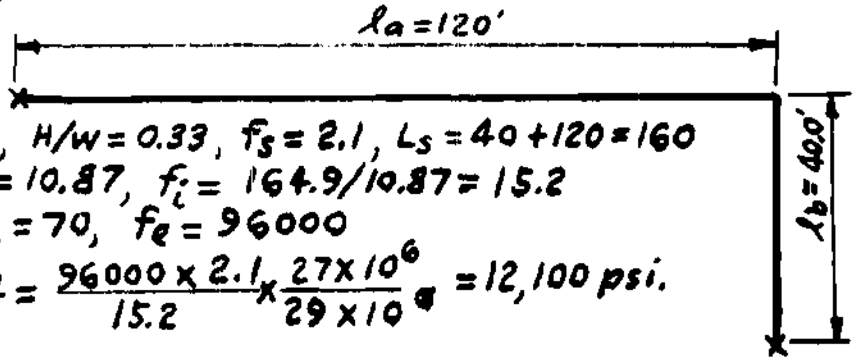
$$S_b = \frac{E_h}{E_c} \beta K_b e \frac{D}{\ell_c} = \frac{27 \times 10^6}{29 \times 10^6} \times 1.64 (9.3) \frac{642\ 6.625}{20}$$

$S_b = 3{,}030$ psi.

Figure 6. Three-Dimensional Bend, Type 3, Method B

Stress Analysis for 6" Schedule 80, ASTM A-53 Grade B Pipe
Design Conditions: 200 psig., 550°F.

Type 1, Method C
Tube Turns (27)



$H = 40,\ W = 120,\ H/W = 0.33,\ f_s = 2.1,\ L_s = 40 + 120 = 160$

$L_r = 4.9,\ D_r = 10.87,\ f_i = 164.9/10.87 = 15.2$

$T_n = 550,\ T_c = 70,\ f_e = 96000$

$$S_E = \frac{f_e f_s}{f_i} \times \frac{E_h}{E_c} = \frac{96000 \times 2.1}{15.2} \times \frac{27 \times 10^6}{29 \times 10^6} = 12{,}100 \text{ psi.}$$

Figure 7. Angle Bend, Type 1, Method C

Type 2, Method C
Tube Turns (28)
Graphical-Analytical

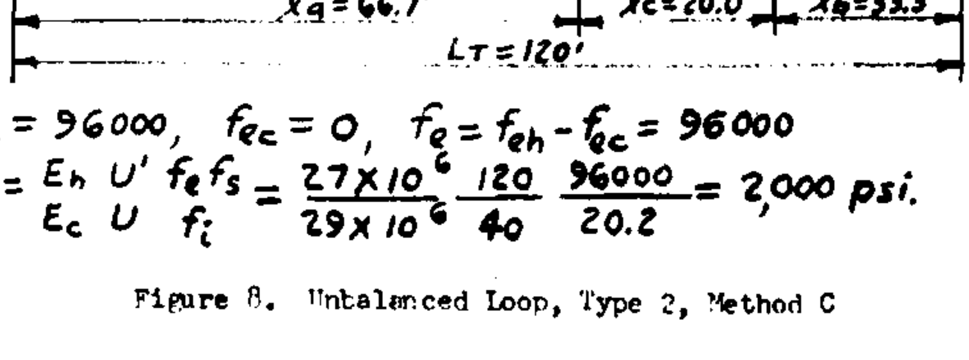
$D_r = 10.87,\ L_r = 4.9,\ U' = 120$

$U = 40.0,\ H = 40,\ W = 20$

$W/U = 0.50,\ H/U = 1.00,\ f_s = 0.154$

$n = 3.8,\ L_s = 200,\ nL_r = 19.6$

$L = 219.6,\ f_i = L/D_r = 20.2$



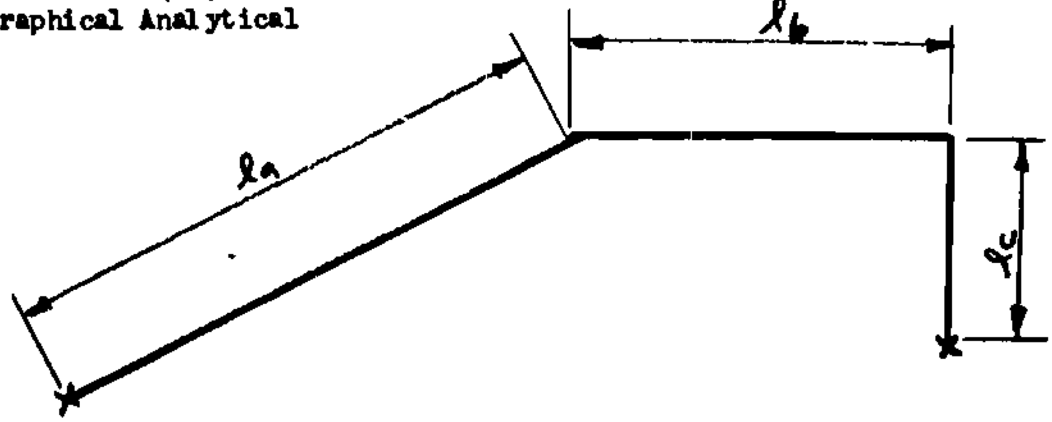
$f_{eh} = 96000,\quad f_{ec} = 0,\quad f_e = f_{eh} - f_{ec} = 96000$

$$S_E = \frac{E_h}{E_c}\,\frac{U'}{U}\,\frac{f_e f_s}{f_i} = \frac{27 \times 10^6}{29 \times 10^6}\,\frac{120}{40}\,\frac{96000}{20.2} = 2{,}000 \text{ psi.}$$

Figure 8. Unbalanced Loop, Type 2, Method C

Stress Analysis for 6" Schedule 80, ASTM A-53 Grade B Pipe
Design Conditions: 200 psig., 550°F.

Type 3, Method C
Tube Turns (29)
Graphical Analytical

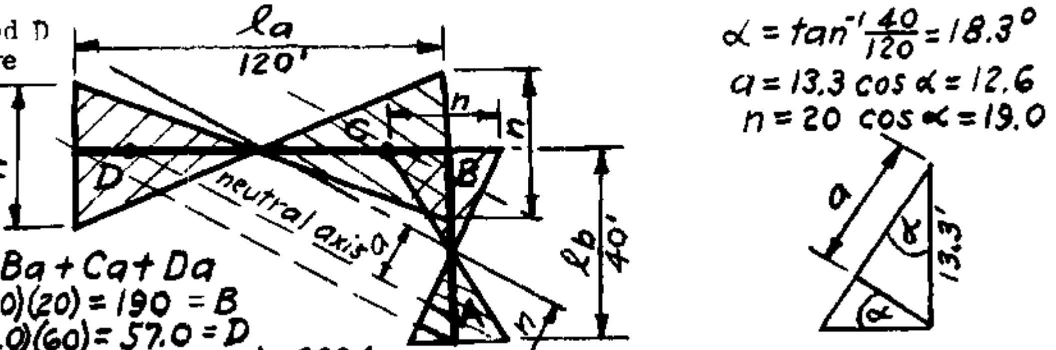


(The Tube Turns Graphical-Analytical Method is not applicable to Type 3 Bend unless hypothetical anchors are placed between the two end anchors. This results in an error of 10-50%. Due to this inaccuracy, the method will not be used in this investigation).

Figure 9. Three-Dimensional Bend, Type 3, Method C

Stress Analysis for 6" Schedule 80, ASTM A-53 Grade B Pipe
Design Conditions: 200 psig., 550°F.

Type 1, Method D
National Valve
& Mfg.Co.(30)
Graphical



$\alpha = \tan^{-1}\frac{40}{120} = 18.3°$

$a = 13.3 \cos\alpha = 12.6$

$n = 20 \cos\alpha = 19.0$

$M = Aa + Ba + Ca + Da$

$A = \frac{1}{2}(19.0)(20) = 190 = B$

$C = \frac{1}{2}(19.0)(60) = 57.0 = D$

$M = 12.6(190 + 190 + 57.0 + 57.0) = 6224$

$P = \Delta EI/1728\,M,\quad P_c = P\,T/t = \left[\frac{4.11 \times 120 \times 27 \times 10^6 \times 40.49}{100 \quad 1728\,(6224)}\right]\frac{160}{159.1} = 505\ lb.$

$S_b = 6\,P_c RD/I = \frac{6\,(505)\,19.0\,(6.625)}{40.49} = 9{,}400\ psi.$

Figure 10. Angle Bend, Type 1, Method D

Type 2, Method D
National Valve
& Mfg. Co.(31)
Graphical

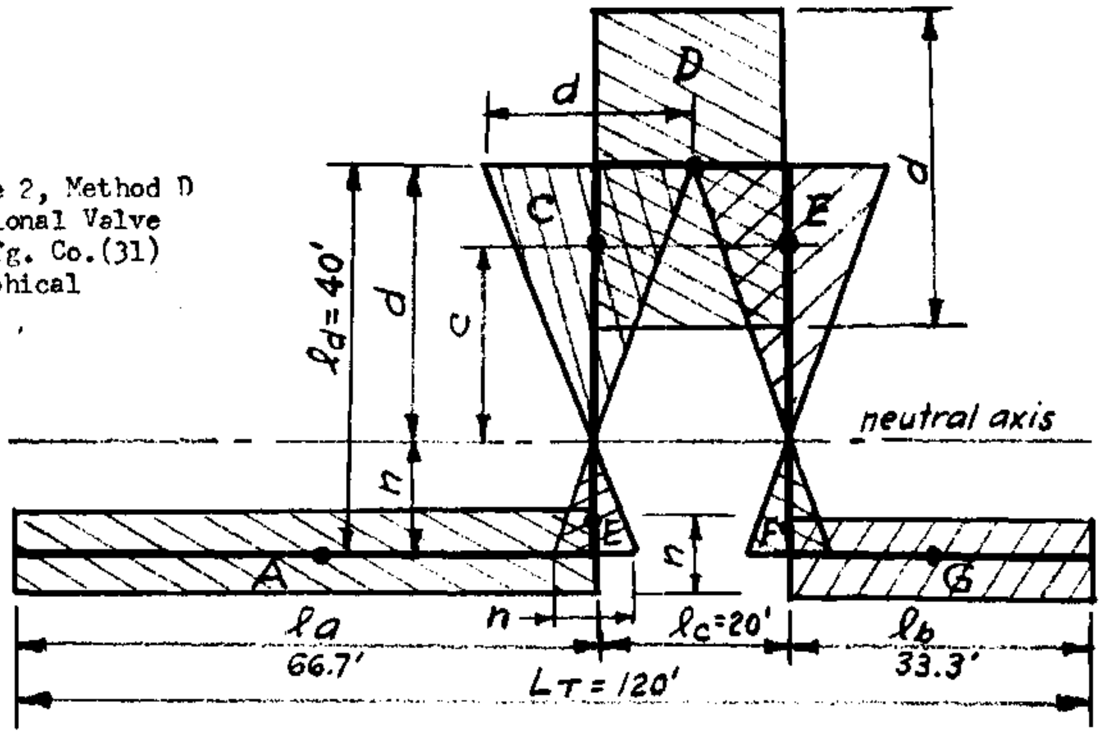


Figure 11. Unbalanced Loop, Type 2, Method D

THERMAL STRESS CALCULATIONS
UNBALANCED LOOP, TYPE 2, METHOD D
NATIONAL VALVE AND MFG. CO., GRAPHICAL
REFERENCE FIGURE 11

Bending Moment = M = An + Bb + Cc + Dd + Ee + Fb + Gn

$$n = \frac{\ell_c \ell_d + \ell_d^2}{2\ell_d + L_T} = \frac{20(40) + (40)^2}{2(40) + 120} = \frac{800 + 1600}{80 + 120} = 12.0'$$

$d = 40 - n = 40 - 12 = 28.0$

$d - c = \frac{40}{3}$, $c = d - 13.3 = 28.0 - 13.3 = 14.7$

$e = c = 14.7$, $n - b = \frac{12.0}{3} = 4.0$

$b = n - 4.0 = 12.0 - 4.0 = 8.0$

$A = \ell_a n = 66.7(12.0) = 790.4$

$B = \frac{1}{2} n^2 = \frac{1}{2} (12)^2 = 72.0$

$C = \frac{1}{2} d^2 = \frac{1}{2} (28.0)^2 = 392$

$D = \ell_c\, d = 20(28.0) = 560$

$E = C = 392$

$F = B = 72.0$

$G = \ell_b\, n = 33.3(12.0) = 399.6$

Bending Moment = 790.4(12.0) + 72.0(8.0) + 392(14.7)
+ 560(28.0) + 392(14.7) + 72.0(8.0)
+ 399.6(12.0)

Bending Moment = 9,485 + 576 + 5,762
+ 15,680 + 5,762 + 576 + 4,795

Bending Moment = 42,636 lb.ft.

THERMAL STRESS CALCULATIONS
UNBALANCED LOOP, TYPE 2, METHOD D
NATIONAL VALVE AND MFG. CO., GRAPHICAL
REFERENCE FIGURE 11

$$P = \frac{\Delta E\ I_p}{1728M} = \frac{\frac{4.11}{100} \times 120 \times 27 \times 10^6 \times 40.49}{1,728 \times 42,636}$$

$P = 73.4$ lbs. $\qquad T = 66.7 + 40 + 20 + 40 + 33.3$

$P_c = \frac{PT}{t}$ $\qquad T = 200$ ft.

$P_c = \frac{73.4 \times 200}{196.4} = 74.5$ lbs. $\qquad t = 200 - 4\ \frac{T \times 9 \text{ ft.}}{4 \times 12} = 200 - 2.4 = 197.6$ ft.

Maximum Bending Stress occurs at points t and u, which are farthest from the neutral axis.

$$\text{Maximum Bending Stress} = \frac{6\ P_c\ RD}{I} = S_b$$

$$S_b = \frac{6(74.5)(28.0)(6.625)}{40.49} = 2,060 \text{ psi.}$$

Stress Analysis for 6" Schedule 80 ASTM A-53, Grade B Pipe,
Design Conditions: 200 psig., 550°F.

Type 3, Method D - National Valve & Mfg. Co.
Graphical (32)

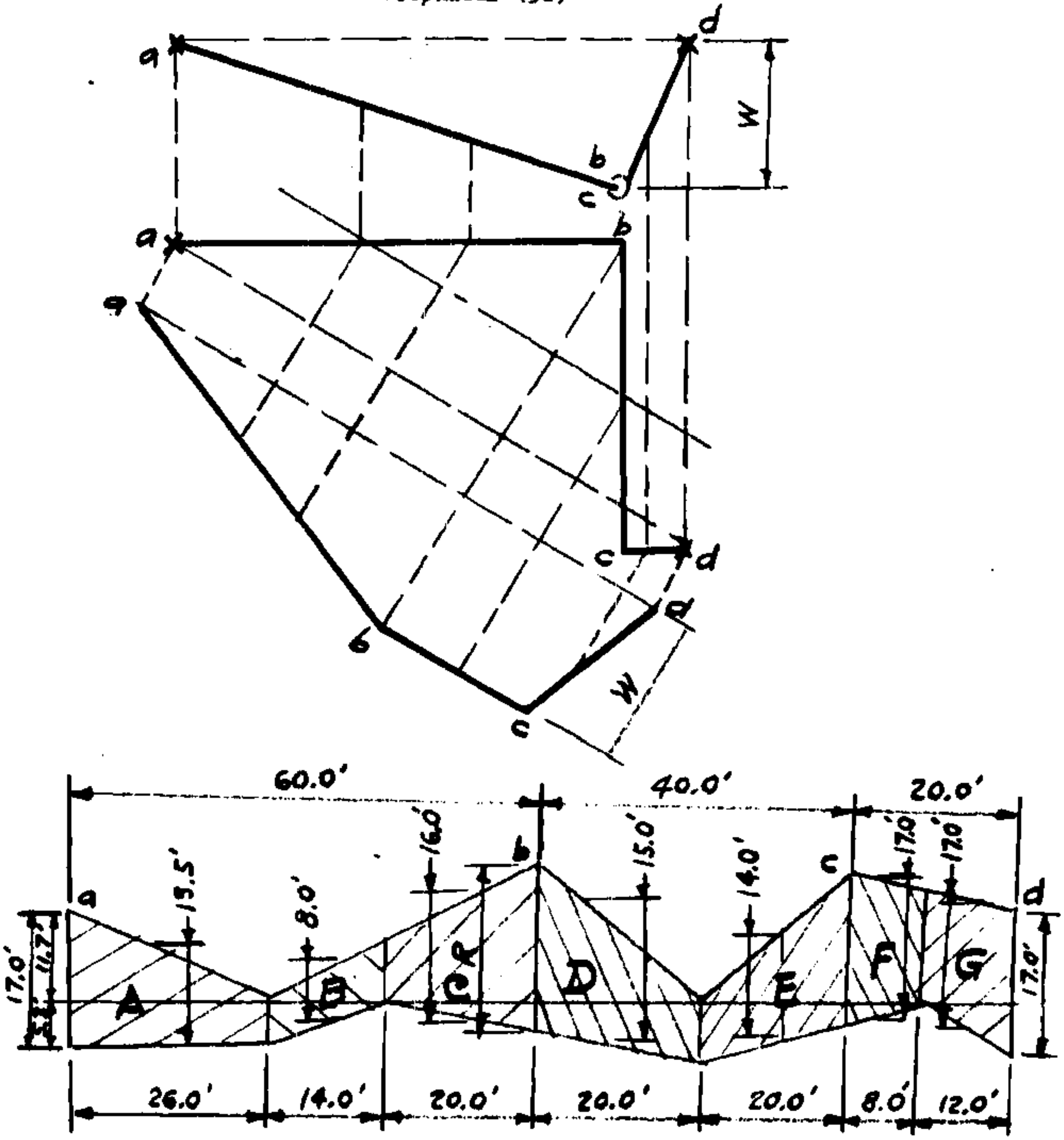


Figure 12. Three-Dimensional Bend, Type 3, Method D

THERMAL STRESS CALCULATIONS
THREE-DIMENSIONAL BEND, TYPE 3, METHOD D
NATIONAL VALVE AND MFG. CO., GRAPHICAL
REFERENCE FIGURE 12

Section	(M) Area	(N) Moment Arm	M = Moment = (M) x (N)
A	1/2(17.0+5.3)26.0 = 290	13.5	3,915
B	1/2(5.3+8.0)14.0 = 93.1	8.0	745
C	1/2(8.0+22.0)20.0 = 300	16.0	4,800
D	1/2(22.0+8.0)20.0 = 300	15.0	4,500
E	1/2(8.0+19.0)20.0 = 270	14.0	3,780
F	1/2(19.0+16.0)8.0 = 140	17.0	2,380
G	1/2(14.0+17.0)12.0 = 372	17.0	6,324
			26,444

$$P = \frac{\Delta EI}{1728\ M}$$

$$P = \frac{3.12 \times 27 \times 10^6 \times 40.46}{1728\ (26,444)} = 74.5 \text{ lb.}$$

$$P_c = P \frac{T}{t} = 74.5 \frac{120.0}{118.2} = 75.4 \text{ lb.}$$

$$S_b = \frac{6\ P_c\ RD}{I_p} = \frac{6(75.4)(22.0)(6.625)}{40.49} = 1,640 \text{ psi.}$$

Variations of Results.--From Table 1 the tabulated results indicate a variation of approximately 26 per cent between high and low values for the angle bend, and a maximum variation from the arithmetic average of 12 per cent.

For the unbalanced loop, the tabulated values show a maximum variation of 26 per cent from the arithmetic average, and a maximum range in values of 50 per cent.

Table 1 indicates that two out of three values were within 30 per cent of their average, while the other value was more than twice the average value.

CHAPTER V

CONCLUSIONS

The chemical engineer has available a number of different methods for use in determining pipe stresses in piping. These methods have been grouped into four basic types: analytical, standardized shapes, graphical-analytical, and graphical. Each of these methods produces a different value for the stresses. However, there is a similarity in the general results obtained.

The scope of this study is confined to a description, illustration, and comparison of a typical method for each type of analysis, and therefore a detailed comparison between these methods on many piping configurations in order to ascertain the relative accuracy of each method has not been made.

It was observed that the analytical and graphical methods afforded more opportunity for a close check to be made of all components of the analysis, while the standardized shapes and graphical-analytical methods required the use of values which could not be readily checked.

Of the methods used, the analytical method required the longest time to complete the analysis. The graphical system was next in time requirement, followed by graphical-analytical and standardized shapes.

CHAPTER VI

RECOMMENDATIONS

It is recommended that the designer of piping for chemical and other industrial plants become familiar with at least one method of stress analysis in each of the four types of analysis covered by this study, and use at least two methods in analyzing piping for thermal stresses in order to have a check on the results obtained. For example, the analytical method might be used as the first type, and the graphical type could then be used to obtain a check on the results.

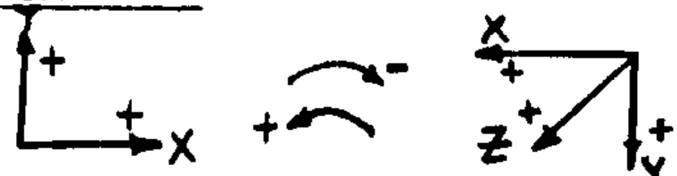
Considerable practice appears necessary in order for the designer to gain a feeling of confidence in the use of such methods of analysis. When two methods have been used and a wide variation in results has been obtained, a third method should be used to more accurately ascertain the most accurate value.

It is also recommended that adequate attention be paid to the accurate determination of stresses in piping in the early phases of the design of plants so that this important engineering work may be incorporated in a successful design.

APPENDIX

DERIVATION OF ANALYTICAL METHOD (12)

A. Sign Conventions:

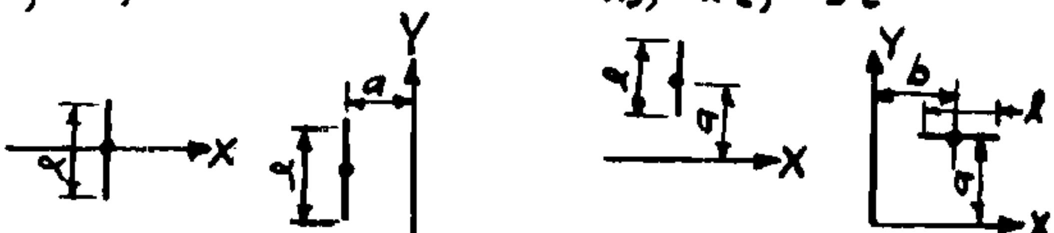


Single Plane   Moment   Two Planes

General Rule: for one end of pipe bend fixed and one end free, a force acting in the positive direction will move the free end in that direction.

B. Inertia:

Axis	Moment of Inertia	Product of Inertia
X	I_x	—
Y	I_y	—
Z	I_z	—
X-Y, X-Z, Y-Z		I_{xy}, I_{xz}, I_{yz}



X-Axis   Y-Axis   X-Axis   X,Y Axes

$I_x = \frac{l^3}{12}$ $I_y = la^2$ $I_x = \frac{l^3}{12} + la^2$ $I_{xy} = lab$

APPENDIX

DERIVATION OF ANALYTICAL METHOD (continued)

C. Force vs. Deflection:

Maxwell's Law of Reciprocity:

Deflection ΔX produced by F_X
$=$ Deflection Δy by F_X in X direction:

$F_x \delta_{xx} + F_y \delta_{yx} = \Delta X$, $\quad F_x \delta_{xy} + F_y \delta_{yy} = \Delta y$

Deflection is proportional to I by use of proportionality constant $1/EI_p$.

$\delta_{xx} = I_x/EI_p$, $\delta_{yy} = I_y/EI_p$, $\delta_{xy} = I_{xy}/EI_p$

Substituting:

$$F_X I_X/EI_p + F_y I_{xy}/EI_p = \Delta X$$
$$F_y I_{xy}/EI_p + F_y I_y/EI_p = \Delta y$$

Transforming and using sign conventions:

$$F_X I_X - F_y I_{xy} = \Delta X \, EI_p$$
$$-F_y I_{xy} + F_y I_y = \Delta y \, EI_p$$

For Three-Dimensional Bends:

Movements in X, Y, Z directions $= F_X I_X/EI$, $F_y I_{xy}/EI$, $F_z I_{xz}/EI$

Equating Σ movements and using sign conventions:

$$F_X I_X - F_y I_{xy} - F_z I_{xz} = \Delta X \, EI_p \quad \text{(X-direction)}$$
$$-F_X I_{xy} + F_y I_y - F_z I_{yz} = \Delta y \, EI_p \quad \text{(Y-direction)}$$
$$-F_X I_{xz} - F_y I_{yz} + F_z I_z = \Delta Z \, EI_p \quad \text{(Z-direction)}$$

APPENDIX

Equations for determining the value of forces from the analytical method (12) are as follows:

Table 2. Calculation of Forces
Single Plane Piping
Constant Cross Section

Force	Direction	Equation
F_x	Horizontal	$F_x = \dfrac{I_y(\Delta x\ EI_p) + I_{xy}(\Delta y\ EI_p)}{I_x I_y - I_{xy}^2}$
F_y	Vertical	$F_y = \dfrac{I_x(\Delta y\ EI) + I_{xy}(\Delta x\ EI)}{I_x I_y - I_{xy}^2}$

Table 3. Calculation of Forces
Three-Dimensional Piping
Constant Cross Section

Force	Direction	Equation
F_x	Horizontal	Simultaneous equations:
F_y	Horizontal	$F_x I_x - F_y I_{xy} - F_z I_{xz} = \Delta x\ EI_p$
F_z	Vertical	$-F_x I_{xy} + F_y I_y - Z\ I_{yz} = \Delta y\ EI_p$
		$-F_x I_{xz} - F_y I_{yz} + F_z I_z = \Delta z\ EI_p$

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